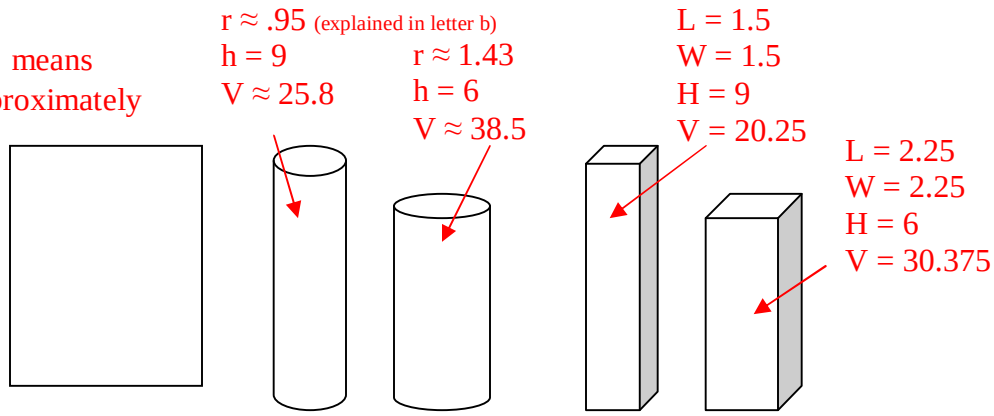
Quiz Review answer key

1. Jackie has a 6 by 9 inch sheet of paper. She wants to use the paper, without cutting it, to make a container with the greatest possible volume. (She will make the top and bottom from another sheet of paper.) She thought of rolling the paper to make an open-ended cylinder and realized that there are two ways to do this. Her friend Renate suggests folding the paper to make a rectangular prism with square ends. Jackie points out that there are also two ways to fold the paper to make the sides of a prism with a square base.

≈ means approximately



a. Which of the four containers has the greatest volume? You may want to make models of the containers to help answer this question.

¶ Cylinder, because what we learned in class was that the more sides the base has, if the perimeter is the same, the larger the area of the base becomes. We learned this when we found the area of the triangular, square, pentagonal, and hexagonal prisms as well as with the cylinder. Also the more cubic a shape is the better the volume, but even better is a sphere. A sphere is the best shape for having the least surface area and the most volume so the closer you get to a sphere, the better the shape.

b. What is that container's volume?

¶ $\pi r^2 xh$ is the volume of a cylinder. However we do not know the radius of the cylinder so we need to calculate that first. We do know the circumference of the cylinder as it is the long side of the sheet which equals 9 in. Take $9/\pi$ = diameter, then divide by 2 to get to radius, then plug it into the equation.

¶ $9/\pi \approx 2.86$ in for the diameter.

¶ $2.86 / 2 \approx$ **1.43** in for the radius.

¶ π **(1.43)^2 x 6** (6 was the width of the sheet of paper so that equals the height of the cylinder). ≈ **38.5 in^3** (Within 2 is good)

c. How much greater is this container's volume than the volume of the other container of the same height?

¶ The container of the same height would be the short square prism. It would have a height of **6** and a perimeter of **9**. Since the perimeter is **9**, and it is a square, we can divide by **4** to get the side lengths on the square. **9 / 4 = 2.25**. Now to find the volume do L x W x H = 2.25 x 2.25 x 6 = **30.375 in^3**.

¶ **38.5 in^3** (volume of the cylinder) **- 30.375 in^3** (volume of the square prism) = **8.125 in^3** (Within 1 is good)

d. Write a note to Jackie explaining why this container has the greatest volume.

¶ Jackie, The short cylinder is the best container because it is closest to being a sphere and a sphere is the best container overall because it is like a cube without corners. Corners take a lot of material and don't hold a large amount compared to if you had a ball of the same length, width, and height. The ball would hold nearly as much, but use a significantly less amount of material.

In 2-10, use this information: Kola Kola is planning to package their cola in a new party-size can in addition to their regular can.

The regular can has a radius of 2 cm and a height of 10 cm. The party-size can has a radius of 14 cm and a height of *40* cm.

2. How many square centimeters of aluminum are needed to make the regular can? (Assume Kola Kola's cans have flat bottoms and tops.)
   ¶ $\mathbf{2\pi r^2 + 2\pi rh} = 2\pi(2)^2 + 2\pi(2)10 \approx \mathbf{150.8\ cm^2}$
3. How many square centimeters of aluminum are needed to make the party-size can?
   ¶ $2\pi r^2 + 2\pi rh = 2\pi(14)^2 + 2\pi(14)40 \approx \mathbf{4750\ cm^2}$
4. How many cubic centimeters of cola will the regular can hold?
   ¶ $\pi\mathbf{r^2h} = \pi(2)^2 10 \approx \mathbf{125.7\ cm^3}$
5. How many cubic centimeters of cola will the party-size can hold?
   ¶ $\pi r^2 h = \pi(14)^2 40 \approx \mathbf{24630\ cm^3}$
6. How many times greater is the radius of the party-size can than the radius of the regular can?
   ¶ 14 (radius of large can) / 2 (radius of small can) = **The radius is 7 times greater**
7. How many times taller is the party-size can than the regular can?
   ¶ 40 (height of large can) / 10 (height of small can) = **The height is 4 times bigger.**
8. How many times more square centimeters of aluminum are needed to make just the side (not the bases) of the party-size can than to make the side of the regular can? Explain why the party-size can requires this many *times* more aluminum. (Be specific. Don't just say "because it is bigger.")

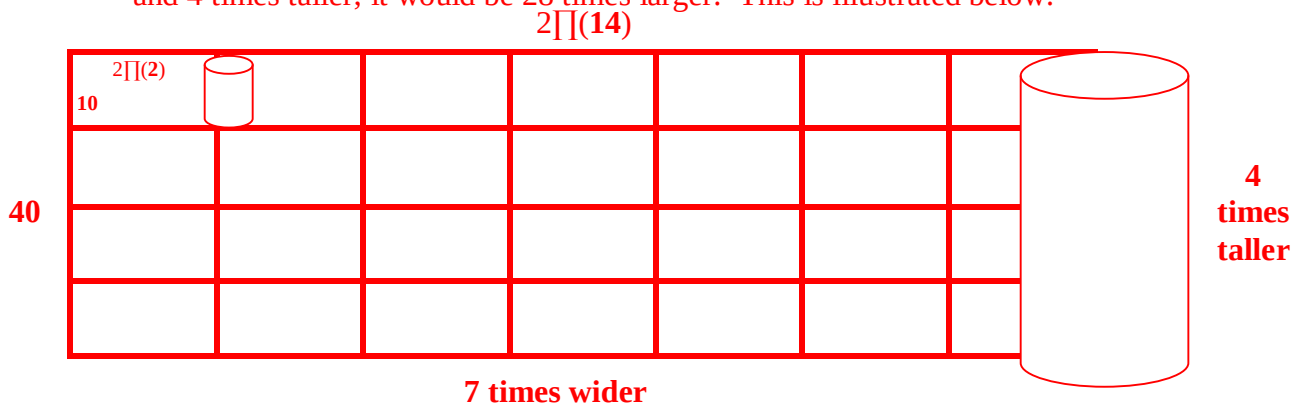
¶ Surface area of a cylinder is $2\pi r^2 + 2\pi rh$

¶ The part before the plus sign is 2 x πr^2 (the area of the circle). We multiply by 2 since there are 2 circles.

¶ The part after the plus sign is $2\pi r$ (circumference of the circle) x h. This gives you the area of the rectangle so you have to perform the equation $2\pi rh$ on both cylinders to find the lateral surface area.

- Small cylinder = $2\pi rh = 2\pi(2)10 \approx$ **125.7 cm^2**
- Large cylinder = $2\pi rh = 2\pi(7)40 \approx$ **1759.3 cm^2**
- 1759.3 cm^2 / 125.7 cm^2 ≈ **28. The lateral surface area of the can is 28 times bigger.**

- The reason that it is 28 times bigger is due to the radius being 7 times bigger and the height being 4 times bigger. Radius being 7 times bigger makes the circumference 7 times bigger. If we think of that rectangle that is being wrapped around the cylinder being 7 times wider and 4 times taller, it would be 28 times larger. This is illustrated below.



9. How many times more cubic centimeters of cola will the party-size can hold than the regular can? Explain why the party-size can holds this many *times* more cola. (Be specific.)
   - The large can will hold about **196** times more.
   - 24630 cm^3 / 125.7 cm^3 ≈196
   - Since the radius is 7 times bigger and the height is 4 times bigger, it is
   - 7 (times longer) x 7 (times wider) x 4 (times taller) which gives you 196 times more volume.

10. If a regular can sells for $0.25 what should the price of the party-size can be if the company wants to base the price on the amount of cola the can will hold?

Since it holds 196 times as much liquid, it would cost 196 times as much or 196 x $0.25 = **$49**